Unhappy Magic Flowers and Binary

Binary is a number system in which each digit can take one of only two values: 0 or 1. Binary allows the computer to encode information in a series of switches that are either on (1) or off (0).

Each binary digit represents a power of two. The first (right-most) digit represents the 1's place, the second digit represents the 2's place, the third represents the 4's place, and so forth. For example, the binary number $10110 = (1 \cdot 2^4) + (0 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0) = 22$ in decimal.

THE DELIVERYMAN PAUSED OUTSIDE of the wizard Marcus's New Athens townhouse. Marcus smiled. He had recently returned from visiting King Fredrick in the capital, and he was waiting for the backlog of missed deliveries. There should be at least five important potion ingredients and a new hat arriving today.

Yet the deliveryman didn't continue toward the door. He stood transfixed, staring at the flowers. After two minutes, Marcus went outside to see if there was a problem.

"Your flowers have changed since yesterday," observed the deliveryman. "I'm sure the one on the right was red yesterday. Today it's blue."

"Those are the same flowers," responded Marcus. "Some of them are sulking today. Stupid flowers."

"Sulking? Do flowers sulk?" the deliveryman asked.

"Actually, it's more of a protest," clarified Marcus. "They are,

of course, magic. They protest whenever it doesn't rain. It's quite aggravating, really. I water them every day, yet they still insist on sulking."

"Huh?" The deliveryman looked back and forth between Marcus and the flowers, trying to make sense of Marcus's statement.

"They protest by changing color to blue. Roses are supposed to be red. You have probably heard all of the poems to that effect, 'Roses are red' and such. But these roses insist on telling me how long they have had to 'suffer' without rain." Marcus gestured angrily at the roses as he spoke.

"They talk to you?" The deliveryman took a step away from Marcus.

"Of course not. They simply change color."

"What does color have to do with when it rained?"

"Well," started Marcus. "they used to all change color together after three days without rain—a sort of mass protest. Then they started to organize. They want to let me know exactly how unhappy they are. So now they count the days."

"Only two are blue," observed the deliveryman. "It hasn't rained all week."

"Nine days, to be exact," corrected Marcus after looking at the flowers. "They use binary."

"Huh?"

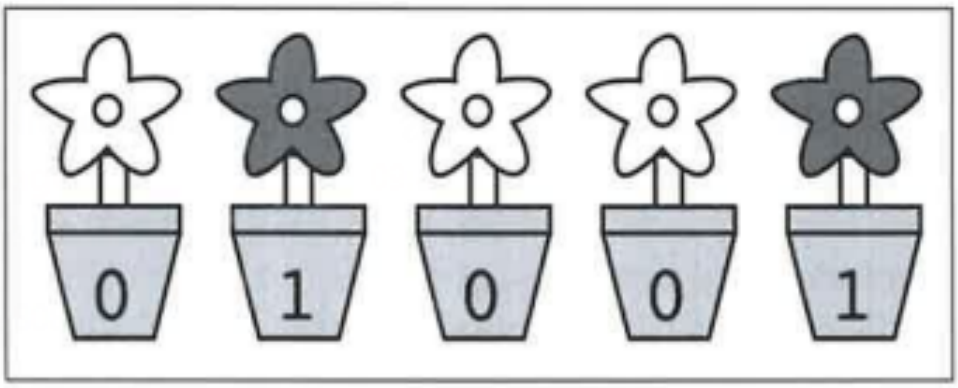
"Red flowers are zero and blue flowers are one," Marcus added.

That explanation did not help. In fact, it seemed to further confuse the deliveryman. However, on the positive side, he no longer looked as though he wanted to run away.

"Binary?" prompted Marcus. "Each flower represents a different digit, and thus a different power of two. The rightmost flower means one (2^0), the one next to it means two (2^1), the one next to that means four (2^2), and so forth. Add up the numbers represented by the blue flowers and you get the total number of days. Right now only the first ($2^0 = 1$) and fourth ($2^3 = 8$) flowers are blue, so it's been $2^0 + 2^3 = 1 + 8 = 9$ days."

The deliveryman looked. Sure enough, the five flowers across

Marcus's porch were: Red Blue Red Red Blue (or 01001).



"Why do they use binary?" asked the deliveryman.

"They tried to spell out the numbers on their petals, and they got too confused. So they had to settle for each flower being either all red or all blue. It turns out that flowers aren't that smart. Binary is a simple enough system for them. If they were smart enough for anything else, do you think they would be complaining to me about the rain? There's nothing I can do about it!" Marcus shouted the last part directly at the flowers.

"But how do they work together?" The more absurd the story got, the more interested the deliveryman became. He leaned in toward the flowers.

"It's really quite simple for them to count in binary," started Marcus. "When it rains, they're all happy and turn red. They effectively reset the counter to 00000. I like those days a lot.

"Then, each morning all of the flowers wake up and decide what color they'll be for the whole day. If it hasn't rained, they increase the count.

After 1 day: Red Red Red Red Blue (00001 = 1)
After 2 days: Red Red Red Blue Red (00010 = 2)
After 3 days: Red Red Red Blue Blue (00011 = 2 + 1 = 3)
After 4 days: Red Red Blue Red Red (00100 = 4)
After 5 days: Red Red Blue Red Blue (00101 = 4 + 1 = 5)

and so on.

"You see, each flower looks to its right in order to decide what to do. If its right-hand neighbor changes from blue to red (1 to 0), then the flower flips its own color. A blue flower changes to red,

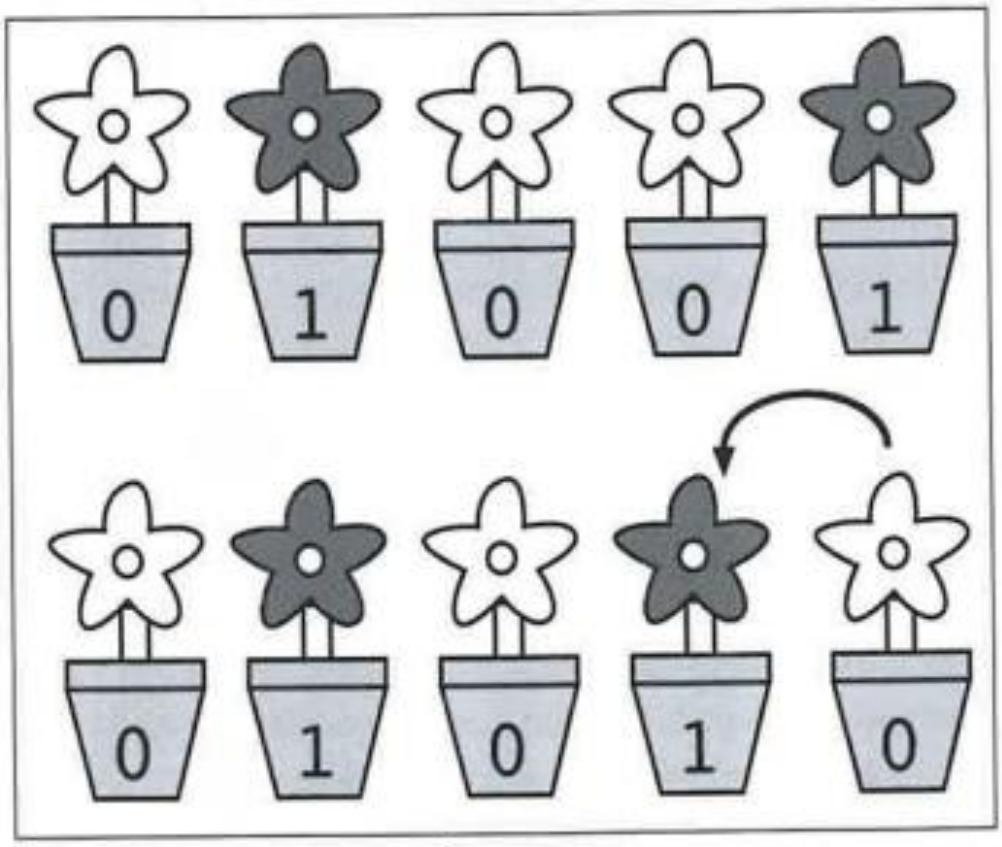
and a red flower changes to blue. This keeps happening until one of the flowers doesn't change from blue to red."

"What about the right-most flower?" asked the deliveryman. "How does it know what to do?"

"Ah. *That one is the instigator*! I'm sure he's the one that started it. Every morning there's no rain, he flips. He's the one that starts the process off. Red to blue to red to blue."

The deliveryman thought about it. "Why does a flower only change when its neighbor flips from blue to red?"

"Think about it the way that you would count with numbers 0-9. When you hit 9, you can't go any further with that digit. So you increase the next digit by one and roll the current digit back to 0. It's like going from 19 to 20 or from 29 to 30. Only here there are exactly two options for each digit, 0 and 1, so things roll over more frequently."



"That system always works?" interrupted the deliveryman.

Marcus tore his attention away from the right-most flower. He suddenly wondered how the discussion had gone from a rant about magic flowers to counting in binary. Did the deliveryman

not have any other deliveries? For that matter, where was the delivery for Marcus?

"Yes. They've already counted out nine days, haven't they?" answered Marcus flippantly.

Then, seeing the look of interest on the deliveryman's face, Marcus returned to his teaching tone. "Consider what happens if doesn't rain tomorrow. The first flower will switch from blue to red, so the second flower will switch from red to blue. The count will go from 01001 = 9 to 01010 = 10."

The deliveryman looked impressed. Marcus couldn't understand why. The flowers were really annoying.